

Domain with Noncompactified Extra Dimensions in Multidimensional Universe with Compactified Extra Dimensions

Dzhunushaliev V.D. *

Freie Universität Berlin, Fachbereich Physik, Arnimallee 14, 14195 Berlin, FRG;

Dept. of Theor. Phys. Kyrgyz State National University

Bishkek, 720024, Kyrgyzstan

Abstract

It is supposed that in our Universe with compactified extra dimensions (ED) the domains exist with noncompactified ED. Such domain can be a wormhole-like solution in multidimensional gravity (MD), located between two null surfaces. With the availability of compactification mechanism this MD domain can be joined on null surfaces with two black holes filled by gauge field. Solution of this kind in MD gravity on the principal bundle with structural group $SU(3)$ is obtained. This solution is wormhole-like object located between two null surfaces $ds^2 = 0$. In some sense these solutions are dual to black holes: they are statical spherically symmetric solutions under null surfaces whereas black holes are statical spherically symmetric solutions outside of event horizon.

Typeset using REVTeX

*E-mail address: dzhun@freenet.bishkek.su

Keywords: multidimensional gravity, gauge field, wormhole, compactification, extra dimension.

Name: Dzhunushaliev Vladimir

Postal address: mcr.Asanbai, d.25, kv.24, 720060, Bishkek, Kyrgyzstan

Telephone: 007 (3312) 46-57-45

E-mail: dzhun@freenet.bishkek.su

I. INTRODUCTION

Currently have been understood that an important feature of modern Grand Unified Theories (GUT) is its multidimensionality (MD). In these theories our Universe is MD Universe with compactified extra dimensions. In 80-th years the MD gravity on fibre bundles with fibres=symmetrical space had been investigated intensively (see, for example, [1]- [4]). In these works had been established that above mentioned MD gravity is equivalent to 4D gravity + gauge field + scalar field. The multidimensionality is a very serious component for almost all recent GUT. In these theories the superfluous extra dimensions (ED) are frozen and contracted to very small sizes. We can suppose, nevertheless, that there are some regions with noncompactified ED in our MD Universe. This can be true near to singularity which make up the pointlike elementary particle and cosmology singularity. In first case such domains with noncompactified ED can be found under event horizon.

Now the cosmological solutions, describing the properties of P-brains, interacting with MD gravity are investigated intensively [5]. Also MD cosmological models with MD spacetime $M = M_0 \times \prod_{i=1}^n M_i$ ($n \geq 1$) are examined under dimensional reduction to 4D-multiscalar fields [6]. In [7] a classification of MD inflationary models investigated.

In this note the spherically symmetric solution in MD gravity is sought, which can be a domain, where ED is remain in ON-state even though compactification have taken place in our MD Universe. These would be regions with very strong gravitational fields.

In this work we examine such a MD gravity on a principal bundle. The importance of such theory follows from the following theorem [1]- [2]:

Let G group be the fibre of a principal bundle. Then there is the one-to-one correspondence between G -invariant metrics on the total space \mathcal{X} and the triplets $(g_{\mu\nu}, A_\mu^a, h\gamma_{ab})$, where $g_{\mu\nu}$ is

Einstein's pseudo - Riemannian metric, A_μ^a is a gauge field of the G group, and $h\gamma_{ab}$ a symmetric metric on the fibre. In this case we can write down the MD Ricci scalar $R^{(MD)}$ in the following way:

$$R^{(MD)} = R^{(4)} + R^{(G)} - \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - d\partial_\mu(g^{\mu\nu}h^{-1}\partial_\nu h) - \frac{d(d+1)}{4h^2}\partial_\mu h\partial^\mu h, \quad (1)$$

where $R^{(4)}$ is Ricci scalar of Einstein's 4D spacetime; $R^{(G)}$ is Ricci scalar of the gauge group G , $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - f_{abc}A_\mu^b A_\nu^c$ is the gauge field strength, d is the dimension of the gauge group, ∇_μ is the covariant derivative on 4D spacetime and f_{abc} are the structural constants of given gauge group, h is linear size of fibre and tensor Ricci $R_{\mu\nu} = R_{\mu\nu\alpha}^\alpha$.

This theorem tell us that the nondiagonal components of MD metric can be continued to 4D region as physical gauge fields (electromagnetic, U(2) or SU(3)). Of course we must have the compactification mechanism on the boundary between domains with compactified and noncompactified ED. We don't discuss this phenomenon but assume its existence.

II. THE GRAVITY EQUATION

We note that the metric on the fibre has the following form:

$$ds_{\text{fiber}}^2 = h(x^\mu)\sigma^a\sigma_a, \quad (2)$$

where conformal factor $h(x^\mu)$ depends only on spacetime coordinates x^μ , here $\mu = 0, 1, 2, 3$ are the spacetime indexes, $\sigma_a = \gamma_{ab}\sigma^b$; γ_{ab} is the euclidean metric and $a = 4, 5, \dots \dim G$ index on fibre (internal space). This follows from the fact that the fibre is a symmetrical space (gauge group). σ^a are one-forms which satisfy Maurer - Cartan structure equations:

$$d\sigma^a = f_{bc}^a\sigma^b \wedge \sigma^c, \quad (3)$$

where f_{bc}^a is a structural constant of gauge group. Thus, MD metric on the total space can be written in the following view:

$$ds^2 = ds_{fibre}^2 + 2G_{A\mu}dx^A dx^\mu, \quad (4)$$

where $A = 0, 1, \dots, \dim G$ is multidimensional index on the total space.

Hence we have only following independent degrees of freedom: conformal factor $h(x^\mu)$ and MD metric $G_{A\mu}$. Varying with respect to these variables leads to the following gravity equations:

$$R_{A\mu}^{(MD)} - \frac{1}{2}G_{A\mu}R^{(MD)} = 0, \quad (5)$$

$$R^{(MD)a}{}_a = 0. \quad (6)$$

These equations are vacuum Einstein's MD equations for gravity on the principal bundle.

We can also write down these equations in 4D form using the Lagrangian (1):

$$R_{\mu\nu}^{(4)} - \frac{1}{2}g_{\mu\nu}R^{(4)} = T_{\mu\nu}, \quad (7)$$

where $T_{\mu\nu}$ is energy-impulse tensor for gauge field A_μ^a and scalar field $h(x^\mu)$.

III. SU(3) SPHERICALLY SYMMETRIC SOLUTION

Ansatz for SU(3) potential we take the same as for SU(3) black hole in 4D gravity [8]:

$$A_0 = T_a \frac{x^a}{r} \varphi(r) + \lambda_8 w(r), \quad (8)$$

$$A_i = T_a \epsilon_{aij} \frac{x^j}{r^2} (1 - f(r)) \quad (9)$$

for embeddings of the SU(2) group into SU(3) with the following choices of generators T_a in terms of Gell - Mann matrices:

$$T_a = \frac{1}{2}(\lambda_1, \lambda_2, \lambda_3) \quad (10)$$

this is for isospin 1/2. And for isospin 1:

$$A_0 = T_a \frac{x^a}{r} \varphi(r) + \left(\frac{x^\alpha x^\beta}{r^2} - \frac{1}{3} \delta_{\alpha\beta} \right) w(r), \quad (11)$$

$$A_i = T_a \epsilon_{aij} \frac{x^j}{r^2} (1 - f(r)) + (\epsilon_{is\alpha} x_\beta + \epsilon_{is\beta} x_\alpha) \frac{x^s}{r^3} v(r), \quad (12)$$

here $\alpha, \beta = 1, 2, 3$ are indexes of matrix. Generators T_a for isospin 1 are following:

$$T_a = (\lambda_7, -\lambda_5, \lambda_2). \quad (13)$$

4D metric we search in following wormhole - like form:

$$ds^2 = e^{2\nu(r)} dt^2 - dr^2 - a^2(r) (d\theta^2 + \sin^2 \theta d\phi^2). \quad (14)$$

This 4D metric and gauge field A_μ^a correspond to following MD metric:

$$ds^2 = e^{2\nu(r)} dt^2 - r_0^2 e^{2\psi(r)} \sum_{a=1}^8 \left(\sigma^a - A_\mu^a(r) dx^\mu \right)^2 - dr^2 - a^2(r) (d\theta^2 + \sin^2 \theta d\phi^2). \quad (15)$$

For obtaining the field equations we write down the Euler equations for Lagrangian $\sqrt{-G} R^{(MD)}$ after substitution gauge field (8) - (9) and 4D metric (14). As we consider the vacuum Einstein's equations, we must write down $R^{(MD)} = 0$ equation in addition. For simplicity we examine the case $\varphi(r) = 0, f(r) = 1$. Finally after some transformation we have the following system of equations:

$$\nu'' + \nu' \left(\frac{a'}{a} + \nu' + 8\psi' \right) - \frac{1}{2} r_0^2 \exp(2\psi - 2\nu) w'^2 = 0, \quad (16)$$

$$\frac{a''}{a} - 2 + \frac{a'}{a} (\nu' + 8\psi') = 0, \quad (17)$$

$$8\psi'' + 8\psi' \left(\frac{a'}{a} + \nu' + 8\psi' \right) + \frac{1}{2} r_0^2 \exp(2\psi - 2\nu) w'^2 - 48 \frac{\exp(-2\psi)}{r_0^2} = 0, \quad (18)$$

$$-2 \frac{a'}{a} (\nu' + 8\psi') - 16\psi' \nu' - 56\psi'^2 + \frac{2}{a} - \frac{a'^2}{2a^2} - \frac{1}{2} r_0^2 \exp(2\psi - 2\nu) w'^2 + 24 \frac{\exp(-2\psi)}{r_0^2} = 0, \quad (19)$$

$$\left(\exp(10\psi - \nu) a w' \right)' = 0, \quad (20)$$

where (\prime) means the derivative with respect to r . The last equation (20) is “Yang - Mills” equation (nondiagonal Einstein’s equation) which has the following solution:

$$w' = \frac{q}{ar_0} \exp(\nu - 10\psi), \quad (21)$$

where q is an integration constant (“color charge”). We consider the simplest case when “color charge” q and/or size of extra dimension r_0 is very big:

$$\exp(8\psi) \ll \frac{qr_0}{a}. \quad (22)$$

Then we have the following approximate equations:

$$\nu'' + \nu' \left(\frac{a'}{a} + \nu' + 8\psi' \right) - \frac{q^2}{2r_0^2} \exp(-18\psi) = 0, \quad (23)$$

$$\frac{a''}{a} - 2 + \frac{a'}{a} (\nu' + 8\psi') = 0, \quad (24)$$

$$8\psi'' + 8\psi' \left(\frac{a'}{a} + \nu' + 8\psi' \right) + \frac{q^2}{2r_0^2} \exp(-18\psi) = 0, \quad (25)$$

$$-2\frac{a'}{a} (\nu' + 8\psi') - 16\psi'\nu' - 56\psi'^2 + \frac{2}{a} - \frac{a'^2}{2a^2} - \frac{q^2}{2r_0^2} \exp(-18\psi) = 0, \quad (26)$$

These equations have the following solution

$$\nu = -8\psi, \quad (27)$$

$$\exp(9\psi) = \frac{q}{2a_0} \cos\left(\frac{3}{2} \arctan\left(\frac{r}{a_0}\right)\right), \quad (28)$$

$$w = \frac{8}{3} \frac{a_0}{qr_0} \tan\left(\frac{3}{2} \arctan\left(\frac{r}{a_0}\right)\right), \quad (29)$$

where $a_0 = a(0)$. In this case the condition (22) leads to the following:

$$\frac{qr_0^9}{a_0^{10}} \gg 1. \quad (30)$$

Let us define r_H by which we will have the null surfaces $ds^2 = 0$:

$$g_{tt}(r_H) = \exp(2\nu(r_H)) - r_0^2 \exp(2\psi(r_H)) \sum_{a=1}^8 \left(A_t^2(r_H) \right) = 0. \quad (31)$$

It is easy to see that this may be satisfied by $r_H = \pm a_0$. By $r = 0$ we have the throat of wormhole, hence we can say that this solution is a wormhole - like object located between two null surfaces $r_H = \pm a_0$.

Note that earlier the similar solutions were obtained for U(1) (5D gravity) [9] and for SU(2) (7D gravity) [10] cases.

IV. DISCUSSION

Thus, we can say that all MD gravity on the principal bundles with physical significant structural (gauge) groups U(1), SU(2) and SU(3) have the spherically symmetric wormhole - like solutions located between two null surfaces. In these theories gravity acts on whole total space of principal bundle (maybe this is a situation near gravitational singularity). If we suppose that the compactification mechanism of ED exists in nature then we can sew these wormhole - like objects with corresponding black holes: wormhole-like solutions for 5D, 7D and 12D gravities with Reissner - Nordström's black hole, SU(2) and SU(3) sphalerons respectively. The compactification mechanism based on algorithmical viewpoint and relevant for such joining in [11] is considered. Such composite wormholes will connect two asymptotically flat spaces. For U(1) case this is a model of electrical charge [12] without charge proposed by J.Wheeler.

V. ACKNOWLEDGMENTS

I am very grateful prof.H.Kleinert for invitation to Freie Universität Berlin and DAAD for stipendium.

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